

Calculation vs. Context

Quantitative Literacy and
Its Implications for Teacher Education

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Quantitative Literacy and Its Implications for Teacher Education

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Arguing with Numbers: Teaching Quantitative Reasoning through Argument and Writing

Neil Lutsky
*Carleton College**

Numbers [are] the principal language of public argument
— “More or Less,” BBC Radio Programme (2007)

This chapter argues for numbers and for an approach to teaching quantitative reasoning that involves secondary and post-secondary teachers representing diverse subject matters and disciplines. My arguments are organized around the following propositions:

(i) *Strengthening students’ quantitative reasoning is an imperative of contemporary general education.* This critical need is insufficiently addressed across secondary and post-secondary curricula. One reason is that current justifications for quantitative literacy across the curriculum do not appear relevant to what teachers are charged with doing or believe themselves prepared to do in their classes. That leads to proposition (ii).

(ii) *A fitting context for quantitative reasoning is argumentation, the construction, communication, and evaluation of arguments.* I argue quantitative

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reasoning is potentially relevant to a wide variety of claims individuals seek to advance in public discourse and will present evidence from a particular sample of arguments in college students' written work in partial support of that assertion. Quantitative reasoning can help students as they construct and evaluate arguments. This is because quantitative reasoning can contribute to the framing, articulation, testing, principled presentation, and public analysis of arguments. But what quantitative reasoning skills are especially useful for the purposes of constructing, communicating, and evaluating arguments? That leads to proposition (iii).

(iii) *The quantitative reasoning habits students need to learn are primarily simple and non-technical.* I seek to elaborate this point by listing 10 quantitative reasoning questions that may help students interrogate arguments or prepare arguments for interrogation. But in what contexts might teachers advance quantitative reasoning skills pegged to basic concerns? That leads to proposition (iv).

(iv) *The teaching of quantitative reasoning across the curriculum might not only model itself on the teaching of writing across the curriculum; it might be intertwined with teaching writing.* I will identify suggestions for teaching students to argue with numbers in their writing. These are based on the outcomes of research my colleagues and I have conducted on student uses, misuses, and missed uses of quantitative reasoning in written work and on resources available to teachers seeking to integrate the teaching of writing and of quantitative reasoning.

Quantitative Literacy in General Education

Why does quantitative literacy merit keen attention in the agenda of secondary and post-secondary education? Lynn Steen and his associates (1997, 2001, 2004) have answered this question in compelling fashion by highlighting how pervasive quantitative information is in contemporary life. Numbers are a staple of accounts of world events (Paulos, 1995), environmental trends and challenges (e.g., Gore, 2006), public policy (e.g., Best, 2001, 2004), financial matters and investing (e.g., Taleb, 2004), consumer choices and advertising (e.g., Seelye, 2006), medical news and health decision-making (e.g., Gigerenzer, 2002), educational assessments (e.g. American Institutes for Research, 2006), economic and technological developments (e.g., Friedman, 2005; Committee on Prospering in the Global Economy of the 21st Century, 2005), science news (e.g., Goldacre, 2005), and everyday issues (e.g., Levitt & Dubner, 2005). As Steen has stated, "The world of the twenty-first century is a world awash in numbers" (1997, p. 1).

As educators we need to draw attention to *why* numbers are so widely used in modern life (e.g., Cohen, 2005; Porter, 1995). We need to show others that numbers can contribute to precision in our thinking, facilitate the public discussion and evaluation of claims, help us grasp the attributes of large and complex phenomena, organize vast domains of information, and help us discover patterns of relationships not readily available to human perception. In sum, numbers are not only important because they are pervasive; they are pervasive because they are important. It is because numbers have both the power to influence and the power to inform that we need to educate citizens to attend to numbers, to understand them, and to think thoughtfully and critically about them.

Recent discussions of the goals of higher education acknowledge the growing significance of quantitative literacy, and credit for that rests, at least in part, with advocates such as Best (e.g., 2004), Madison and Steen (2003), Paulos (1988), Schield (2005), Steen (1997, 2001, 2004), and others. Derek Bok (2006), for example, is promoting a list of broad aims for contemporary undergraduate education, including strengthening communication skills, critical thinking, moral reasoning, responsible citizenship, appreciations of diversity, involvement in a global society, breadth of knowledge, and preparations for work. In the context of his treatment of critical thinking, he notes, "certain basic quantitative methods seem applicable to a wide enough range of situations to be valuable for almost all students" (2006, p. 69). (I would add, in keeping with the arguments of those aforementioned advocates, that quantitative literacy could be seen as equally essential to other educational purposes Bok identifies, such as appreciating diversity, living in a more global society, and preparing for work.) Similarly, a recent report by the Association of American Colleges and Universities (2005), *Liberal Education Outcomes*, suggests that there is "a remarkable consensus on a few key outcomes that all students, regardless of major or academic background, should achieve during undergraduate study" (p. 2). That report specifies quantitative literacy as one of those outcomes (see also the 2007 Association of American of Colleges and Universities report, *College Learning for the New Global Century*). Finally, if the reader prefers a more succinct curricular directive, he or she could do no better than Princeton philosopher K. Anthony Appiah's general education recommendation to contemporary students under the heading: "Learn Statistics. Go Abroad" (Appiah, 2005).

One feature common to current curricular discussions is support for a quantitative literacy across the curriculum approach. This has long been advocated in the quantitative literacy literature (e.g., Orrill, 1997, p. xiii) and has been reiterated in broad treatments of curricular priorities. Bok (2006), for example, suggests that:

...numeracy is not something mastered in a single course. The ability to apply quantitative methods to real-world problems requires a faculty and an insight and intuition that can be developed only through repeated practice. Thus quantitative material needs to permeate the curriculum. (p. 134)

This call for quantitative literacy to be taught across the general education curriculum, as well as across all levels of education (Conference Board of the Mathematical Sciences, 2001), resonates with what educators and psychologists know about conditions that facilitate generalized learning. For example, Halpern and Hakel (2003) conclude that teaching for the transfer (generalization) and long-term retention of knowledge requires learners “to generate responses, with minimal cues, repeatedly over time with varied applications so that recall becomes fluent and is more likely to occur across difference contexts and content domains” (p. 38).

But how can quantitative literacy be taught for the purposes of general education? One response to this is to teach quantitative literacy in mathematics and (a) hope that students have reinforcing encounters with quantitative thinking in other courses, or (b) orient the quantitative mathematics courses themselves to be more broadly problem-based (e.g., Nolan & Speed, 1999). Another response is to teach quantitative literacy in other disciplines that employ quantitative analysis as an investigative tool, such as the social sciences, and to relieve mathematics of the sole or even primary educational responsibility for quantitative literacy. This chapter argues for a third way, one that has the potential to broaden the uses to which quantitative reasoning is put and the places in the curriculum it is taught.

The model it emulates is writing across the curriculum. As David Bressoud wrote in the forward to *Achieving Quantitative Literacy* (Steen, 2004), “Quantitative literacy does not need to be taught only by mathematicians any more than effective writing needs to be taught only by English professors” (p. ix). But however compelling it might be on educational grounds to teach quantitative literacy across the curriculum and however appropriate it might be to do so in meaningful, distributed contexts, there are reasons why it has proven much more difficult to forge quantitative literacy across the curriculum initiatives than writing across the curriculum ones. Writing is a means of expression common to most disciplines, whereas quantitative literacy appears relevant to courses in the social and natural sciences but, with minor exceptions, not elsewhere. Moreover, secondary and post-secondary instructors are more likely to be confident in their abilities to teach writing than quantitative analysis, even if only at a basic level. So key challenges remain: why should

teachers in a variety of subject matters believe quantitative literacy is relevant to what they do and, moreover, why should they believe they possess the ability and background to help students strengthen quantitative reasoning habits of mind? Perhaps these challenges can be met if we reconsider the conventional contextual framing of quantitative literacy.

Quantitative Reasoning in the Context of Argument

The primary thesis of this chapter is that quantitative literacy can be usefully situated in the context of argument, in the presentation of statements supporting claims. In this sense, arguments are not only reasons to take one position or another on a contentious issue but address explicit and even implicit claims about the nature of a phenomenon or the importance of a topic (see, e.g., Fulkerson, 1996; Ramage, Bean, & Johnson, 2007). Teaching students how to identify and find the constituent elements of an argument, how to organize arguments systematically, what kinds of statements support particular arguments effectively, how to present arguments clearly and meaningfully to an audience, how to address their own arguments reflectively, and how to evaluate others' arguments *are* fundamental to education at all levels and in almost all disciplines.

What can quantitative information do for arguments? Among other things, quantitative information may be used to help articulate or clarify an argument, frame or draw attention to an argument, make a descriptive argument, or support, qualify, or evaluate an argument. Quantitative analysis may also influence how arguments are marshaled and how exchanges of arguments are conducted. As Robert Abelson (1995) wrote, "the purpose of statistics is to organize a useful argument from quantitative evidence, using a form of principled rhetoric" (1995, p. xiii). Moreover, such arguments are open to knowledgeable evaluation. According to Theodore Porter (1995), "In practice, objectivity and factuality rarely mean self-evident truth. Instead, they imply openness to possible refutation by other experts" (p. 214). This is one of the signal virtues of quantitative analysis; it contributes to open tests of ideas that can be reported in argument and evaluated by others.

Quantitative reasoning has been linked to argumentation previously, but in the existing literature primarily so with regards to how quantitative results are interpreted (although students also commonly face the challenge of taking word problems and figuring out what statistical procedures might be needed to answer them). There is a wonderful Edward Koren cartoon from *The New Yorker* (December 9, 1974) showing the personified numbers 9, 6, 2, 1, 8, and 4 seated on chairs on stage being introduced by a man at the podium who quips, "Tonight, we're going to let the statistics speak for themselves." Of

course, we all know that the numbers do not speak for themselves; someone advocates a case for the sense the numbers might make. To be sure, that is a significant domain of quantitative reasoning, of arguments about the meaning of numbers that are used in arguments with numbers. The Conference Board of the Mathematical Sciences (2001), for example, repeatedly cites interpretation, “relating the results of data analysis back to original questions and stating conclusions” (p. 87), as a basic task elementary, middle school, and high school teachers of statistics should address. But interpreting the meaning of numbers represents only one way in which we argue with numbers, one in which the numbers themselves are the focus of attention rather than the larger arguments of which they are a part.

What a broader approach to examining the relationship between quantitative reasoning and argumentation might yield became clearer to me and my colleagues at Carleton College as we undertook activities associated with our Quantitative Inquiry, Reasoning, and Knowledge (Quirk) initiative. Two years ago eight faculty and academic support staff met to read and discuss papers submitted as part of student writing portfolios required to meet the College’s writing requirement. We wanted to learn whether and how students used quantitative reasoning in written arguments to help us orient workshops for faculty and academic staff. After this informal inquiry, we began developing a more systematic approach to evaluating student papers for quantitative reasoning using a coding rubric we have since been refining (see Quirk Rubric for the Assessment of Quantitative Reasoning in Student Writing, 2007).

What became clear as we developed the rubric was that there were at least two general ways in which students used quantitative reasoning in written argumentation: peripherally and centrally. Peripheral uses cite numbers to provide details, enrich descriptions, present background, or establish frames of reference. Jane Miller (2004), in *The Chicago Guide to Writing about Numbers*, captured the spirit of peripheral applications of quantitative information when she advised her reader, “Even for works that are not inherently quantitative, one or two numeric facts can help convey the importance or context of your topic” (p. 1). An example of a peripheral use of quantitative information is given in a psychology paper that is centrally concerned with identifying possible psychogenic pain mechanisms but peripherally discusses the incidence of psychogenic pain in an introductory paragraph. Central uses of numbers address a *primary* question, issue, or theme in a paper. An example of a central use of quantitative information is given in a paper for an economics course evaluating the need for quotas on textile and apparel imports from China.

We have been using the rubric to code randomly drawn student papers from the portfolios as “potentially employing quantitative information

peripherally” or as “potentially employing quantitative information centrally” or as “not at all or incidentally potentially involving quantitative information” (see Lutsky & Tassava, in preparation, for details). Over two studies (Lutsky, 2006, Lutsky & Tassava, in preparation), we found that roughly two thirds of all papers assessed, representing a sample of papers from courses across the curriculum, were judged as potentially involving quantitative information. Approximately a third of the entire sample of papers potentially involved quantitative information in a peripheral role and a third potentially involved quantitative information in a central role. (Quantitative reasoning was judged as irrelevant to the remaining third of papers.) The peripheral set included papers from across the curriculum; papers from the social and natural sciences dominated the central set. In addition, we judged that two thirds of the papers for which quantitative information was potentially centrally relevant in fact used quantitative reasoning. However, only 12% of the papers for which quantitative information was potentially peripherally relevant used quantitative reasoning.

What do we take these findings as suggesting? First, we should acknowledge that the sample of papers we considered reflects certain limiting conditions (e.g., selection by students to meet the criteria for portfolio inclusion). Moreover, the relevance of quantitative reasoning was judged by two evaluators sensitive to potential uses of quantitative information. Nonetheless, we would advance two tentative observations: (a) quantitative information is potentially relevant to arguments posed in papers from across the curriculum, and (b) quantitative reasoning is strikingly underutilized for peripheral purposes in papers from across the curriculum. The latter is a key finding: *quantitative reasoning could be employed for peripheral argumentation in writing across the curriculum but currently that is not happening.*

Viewing quantitative reasoning through the lens of argumentation raises new challenges for educators. How can we demonstrate to students when quantitative information may be useful in framing or evaluating arguments? How can we train students to find or generate the quantitative information they might begin to seek? At Carleton we have found it useful to work with college librarians to help instruct students on locating relevant data, evaluating data sources, and checking quantitative information. In other words, quantitative literacy in this context has led to a concern for information literacy.

We have also pursued means of teaching students how quantitative evidence might be presented effectively. For example, Fulkerson (1996) suggested readers would evaluate the substantiation for claims in terms of four criteria, which he labeled using the acronym STAR. The first is Sufficiency, whether there is enough evidence provided. The second is Typicality, whether

the evidence presented is representative. The third is Accuracy, whether the data are true. And the fourth is Relevance, whether the evidence is centrally connected to the claim. Quantitative information can be evaluated *as evidence* in light of these criteria and can also provide the grounds for reasoning about the adequacy of substantiations offered for a claim.

In sum, what I have argued above is that a fitting context for quantitative reasoning is argument. As Max Frankel, the Pulitzer Prize winning former editor of *The New York Times* suggested, “Deploying numbers skillfully is as important to communication as deploying verbs” (1995, p. 24). Offering, evaluating, and discussing arguments are activities that are common to a wide range of subject matters. As teachers endeavor to help students think about what makes arguments clear and effective, and how to construct sound and principled arguments, teachers may, if sufficiently trained, prompted, and informed, come to recognize the important roles that quantitative reasoning may play in argumentation. What we have seen is that quantitative reasoning is potentially relevant in both peripheral and central ways to the presentation of arguments, and that potential peripheral uses of quantitative reasoning are both relevant across the curriculum and sorely lacking. That suggests those of us who promote quantitative reasoning across the curriculum have an opportunity to introduce quantitative issues to our colleagues in a simpler, more accessible way than we have previously emphasized.

Quantitative Reasoning Made Simple and Then More Complicated

What is it that we want to educate students to do quantitatively? Taking the construction and evaluation of arguments as a primary concern and remaining attentive to peripheral uses of quantitative information may lead to a reconsideration and simplification of standard quantitative literacy agendas (e.g., Conference Board of the Mathematical Sciences, 2001, pp. 43-44; Steen, 2001, pp. 15-17), at least at the outset of quantitative education. I am not claiming the changes would be radical, nor do I believe they should be, but I do hope the examples of quantitative opportunities and misinterpretations we highlight will become more accessible, relevant, and meaningful to teachers and students when they first encounter quantitative reasoning.

Consider an example of the kind of shortcoming we often tout, recently labeled by Howard Wainer (2007) as “the most dangerous equation” because ignorance of the equation has led to important misunderstandings of quantitative evidence. This is the equation for the standard deviation of the sampling distribution of the mean (i.e., the standard error). Not understanding that variation is likely to be larger when sample sizes are smaller has led,

Wainer shows, to misattributions of the meaning of extreme outcomes derived from small samples. Essentially, statistical artifacts are taken as meaningful. Insensitivity to the relationship between sample size and variability is common in human cognition, as the well-known work of psychologists Tversky and Kahneman (1974) has documented.

I wish, as Wainer does, that these statistical effects were more widely appreciated. But this is not the kind of understanding that is readily accessible to quantitative novices, who may have little sense of what a standard deviation is or what the sampling distribution of the mean is. I need to make clear that in citing this example, I mean no criticism of Wainer, who, after all, was writing for readers of *American Scientist*. My point is that moderately complex examples of unsound statistical reasoning may not encourage educators to promote quantitative reasoning. Rather, what I think we need are simple examples of how quantitative information may strengthen peripheral and central arguments and straightforward questions that can be asked of quantitative claims.

My own attempt to identify a general education agenda for quantitative reasoning represents a response to the following prompt: *What questions would I most want my students spontaneously posing when they encounter opportunities for quantitative argument or existing quantitative arguments?* I have constructed a list of 10 such questions, which I call QR Questions at the Ready (Lutsky, in preparation). These are rooted in the quantitative literacy literature (e.g., Best, 2001, 2004; Goldacre, 2005; Niederman & Boyum, 2003; Paulos, 1988; Steen, 1997, 2001, 2004), my experiences developing and teaching a seminar for first year students at Carleton (Measured Thinking: Reasoning with Numbers about World Events, Health, Science, and Social Issues), and the readings and discussions my colleagues at Carleton and I have had on students' uses of quantitative reasoning, especially as shown in their writing.

What I have tried to do in the list is to state the 10 framing questions in as general a way as possible. Each question subsumes more specific questions, such as those shown, and many of specific questions point to more technical quantitative procedures and issues. I do not take the list to be comprehensive or the best possible list of 10 questions relevant to reasoning about quantitative claims, but I do hope it will stimulate thinking about how we might make quantitative reasoning more accessible to a broad audience in education and beyond.

Here is the list of ten QR "Questions at the Ready":

1. *What do the numbers show?* How can numerical information be used to establish the context or significance of a topic? What is the magnitude of a phenomenon? How can numbers help describe something more precisely?

Is there numerical evidence to support a claim? What are the exact figures? What do cited numbers mean?

2. *How typical is that?* Is the example or anecdotal evidence representative? What is the central tendency? How typical is the central tendency of the scores as a whole or of the scores in subgroups? What is the base rate? What are the odds of that?
3. *Compared to what?* What is the implicit or explicit frame of reference? What is the unit of measurement? Per what? What is the order of magnitude? What defines the Y-axis?
4. *Are findings those of a single study or source or of multiple studies or sources?* What is the source of the numbers? How reliable is it? Has the source been peer-reviewed? Who is sponsoring the research? How plausible is a claimed outcome in light of back of the envelope calculations? Has the finding been replicated? Is there a literature on the finding? Are there converging conclusions from multiple sources? Can the results of a literature be summarized quantitatively? What do the results of relevant meta-analyses indicate?
5. *How were the main characteristics measured?* How were key variables operationalized? What evidence is there that the measurement procedures were reliable, valid, and otherwise sound ones for the purposes of the study? What meaning and degree of precision does the measurement procedure justify?
6. *Who or what was studied?* What domain is being studied? Who or what was sampled from this domain? How was that sample constituted? Was it random? How equivalent are any samples that are being compared?
7. *Is the outcome of a study anything more than noise or chance?* Is the outcome unlikely to have come about by chance (i.e., statistically significant)?
8. *How large is the result of a study?* How substantial is the result? How practically important is it? What is the effect size?
9. *What was the design of the study?* To what extent does the design support causal inferences? Is the design that of a true experiment? Was an experiment double blind?
10. *What else might be influencing the findings?* What other variables might be affecting the findings? Were those assessed or otherwise controlled for in the research design? What do not we know, and how can we acknowledge uncertainties?

Again, I would not claim that the list is sufficient or that it gracefully parses quantitative reasoning at its joints. Pragmatically and logically the first question is most fundamental. We need to teach students the value of thinking in terms of numbers. We need to encourage them to seek relevant numbers, both when they argue and when they evaluate the arguments of others. That is the foundational habit of mind upon which more sophisticated and technical structures of quantitative reasoning can be built.

Writing as a Locus for Teaching Quantitative Reasoning

The teaching of writing provides an inviting opportunity for addressing quantitative reasoning because “argument pervades writing” (Fulkerson, 1996, p. 2). Key values in writing, such as precision in word selection, clarity of expression, persuasiveness, soundness of supporting scholarship and evidence, logical organization, and appeal to readers may be facilitated by quantitatively informed arguments. Writing also involves active learning as students use and think about numbers. Moreover, writing assignments typically give students time to prepare—research, write, and revise—their work and teachers the time to create the educational scaffolding to strengthen writing with numbers.

One essential way teachers can facilitate quantitative reasoning is to give students writing assignments that invite or require quantitative reasoning. Assignments that call for quantitative analysis centrally may be common in the social and natural sciences or in applied statistics courses. Examples of such assignments from across the curriculum are available at the web site of the Science Education Resource Center (Quantitative Writing, 2007). Deann Leoni (2005) has also developed excellent assignments that integrate mathematics and English and get high school students writing with and about numbers.

A major implication of the finding reported earlier on potential peripheral uses of quantitative information is that more could be done to encourage students to cite relevant numbers to frame and introduce topics. That has led us to promote a simple suggestion to faculty at Carleton. *It is to ask students in writing assignments to use numbers to set an example or case study of primary interest in a paper in its wider context.* You may recognize that this is an instantiation of the second of those QR Questions at the Ready: How typical or representative is this? The question has the virtues of directing students to think in terms of numbers and of requiring them to learn how to find (and possibly evaluate) numbers. Typicality of information may also help a writer and his or her reader think about the extent to which and the ways in which the characteristics of the example should be generalized.

Most of the literature on writing and quantitative reasoning offers suggestions for effective ways to write about numbers. A particularly helpful resource for teachers and students in this regard is *The Chicago Guide to Writing About Numbers* (Miller, 2004). Miller identifies principles for expressing numbers in writing, including seven basic ones. These are: (1) establish the context, (2) choose effective examples and analogies, (3) use an appropriate vocabulary, (4) decide where to present numbers, (5) report and interpret numbers in text, (6) specify the size and direction of associations, and (7) summarize overall patterns. Miller also provides specific writing examples to illustrate poor, better, and best efforts to meet these writing goals.

Other authors have particular concerns about how numbers are represented in words. MacNeal (1994), in *Mathsemantics: Making Numbers Talk Sense*, decries the confusion of events with people. Gigerenzer (2002) discusses how representing risks in terms of “natural frequencies” rather than probabilities enhances public understanding. Niederman and Boyum (2003) and Paulos (1988) discuss means of representing units of measurement or large numbers to make them more accessible to readers.

At Carleton we have identified several recurring problems in student writing with numbers. The first, called the *weasel word problem*, highlights overuse of the terms “many,” “often,” “some,” and others of that ilk in the place of either appropriate caution or numbers. Shafer (2005) neatly skewered a front-page article in *The New York Times* (Story, 2005) suffering from the same problem. A second concern, the *staples problem*, refers to papers in which quantitative information in the form of tables and figures is stapled onto a paper but not interpreted in the text (see also Miller’s principle 5). A third shortcoming, the *comparison problem*, indicates instances in which students cite numbers but do not provide frames of reference that might make those numbers meaningful (see also Question 3 of the 10 QR Questions at the Ready). Finally, we have also noted a *terminology variability problem* in the uses of key quantitative terms. Different academic disciplines socialize students to give words such as “experiment” (see Question 9 of 10 QR Questions) more or less restricted meanings.

Other challenges face the teacher attempting to promote student writing using numbers. One, common to writing, is taking the role of the potential reader. How much information and what form of information will be meaningful to readers? One way I have tried to respond to this question in my first year seminar is to bring student writers face to face with readers. I have done this in service learning projects in which teams of students take data collected by community organizations (e.g., the regional Girl Scouts council, a local bike tour) and prepare reports based on the data. I have had the leaders

of the community groups come to class to discuss with students what would make the reports most useful to their organizations. Another important form of this same problem is addressing the reasonable questions of an informed reader. What questions are readers likely to raise about the quantitative claims (findings) presented in a paper? How can these be anticipated and handled in a written report? Finally, a difficult challenge for all of us who use numbers in writing is stating claims with degrees of certainty appropriate to the state of the evidence. As Robert Kuhn has noted, “the cognitive skill to distinguish among hope, faith, possibility, probability, and certitude are potent weapons in anyone’s political survival kit and can be applied in all areas of life and society” (2003, p. 388).

Coda

In a study at Harvard University, Richard Light (2001) asked undergraduate students to identify the characteristics of “faculty who make a difference.” What is it that those faculty do as educators that, according to student self-reports, has a profound impact? Two of the nine attributes students listed were these: teaching precision in the use of language, and teaching the use of evidence. The arguments presented in this chapter suggest the two are not unrelated to each other and are both potentially intertwined with applications of quantitative reasoning. Can recognizing that transform how teachers in secondary and post-secondary education address quantitative reasoning? That, I believe, is an argument worth testing.

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